Nonlinear Standing Waves

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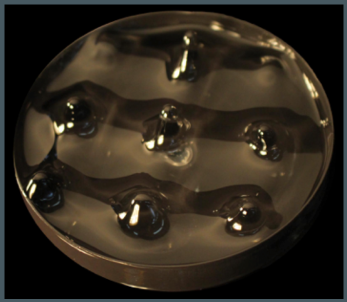
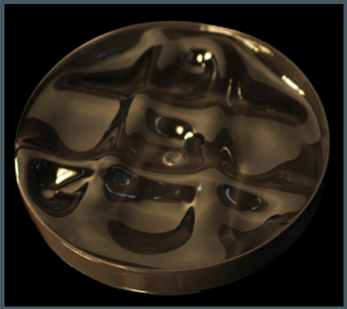




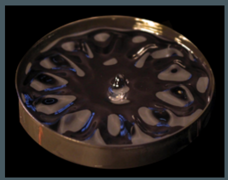
Theory

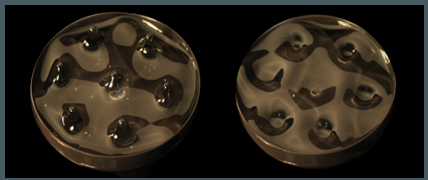
The images above show nonlinear standing waves that oscillate between star- and polygonal-shaped surface features[1]. Nonlinear waves are difficult to analyze mathematically, because the superposition principle does not apply.[2] In this experiment, interesting wave patterns are observed when crests from the inner and outer portions of the dish propagate opposite of each other and undergo a phase shift without superimposing. Peaks in the fluid appear when the counter-propagating, gravity-driven waves break against each other, which occurs periodically at the same location. Interestingly, this particular phenomenon - a five-sided wave geometry - occurs independently of the container size and geometry; the only relevant length scales are the wave amplitude and fluid depth[1].

The patterns do not appear for all oscillatory frequencies or amplitudes, however. In order to visualize these shapes, counter-propagating waves must resonate with each other. Often, the patterns are only observable when the amplitude of the wave is roughly the same order of magnitude as the fluid depth. It is important to realize that the nonlinear patterns are actually long-term instability effects of the motion of the vibrating fluid; minor perturbations would not result in the same images.[3] By varying the oscillation frequency and amplitude, different patterns arise.

Experimental Setup

This visualization replicated the experimental setup used by Rajchenback *et. al 2013.*[1] A 9 cm diameter petri dish was mounted on a vertically oscillating plate which was driven by a Pasco Scientific mechanical wave driver. The dish was filled with approximately 0.5 cm of 100 cSt silicone oil. The frequency and amplitude of the oscillation were adjusted using a function generator to yield different wave patterns. A frequency ranging from 8-10 Hz and amplitude ranging from 1.3-3m5 mm was found to be most successful for achieving the patterns shown here. Rajchenback *et. al.*[1] reported that a 1.85 mm amplitude resulted in waves with five radial axes of symmetry. One challenge in this experiment was manually adjusting the amplitude so that it was large enough to create oscillating standing waves but not so large that the oil splashed out of the petri dish. Images were captured using a Canon Rebel T4i DSLR camera set to ISO 1600 with an aperture of f=5.0 and a shutter speed of 1/1250. Adobe Photoshop Elements 9.0 was used for image post-processing.





Applications

Nonlinear wave interactions are relevant throughout physics, and one important application is oceanic rogue waves. A rogue wave is one in which the amplitude exceeds two times the expected wave height. These can appear on or below the surface of water and their appearance is unpredictable. Researchers have yet to create an accurate mathematical model of rogue waves, but it is expected that the model will be nonlinear.[4,5] It is important to investigate interactions and origins of rogue waves, because they can cause significant damage when they strike cargo ships or offshore structures.[6]

References

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